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ABSTRACT

If healthy mathematical beliefs are to be fostered in students, the teachers' beliefs must also be considered. The purpose of this study was to determine: (1) the impact on interns' (n=137) mathematical beliefs of an exploratory, student-centered mathematics course, "Mathematics for Elementary Teachers Via Problem Solving," (T104) for preservice elementary teachers and (2) whether the T104 style of teaching had a stronger influence on challenging the beliefs of students with high or low levels of mathematics achievement. A Likert-style survey and a short answer survey, as well as nine interviews with preservice techers, were administered. Results of interviews showed four key themes: (1) There is more than one way to solve a problem and some problems have more than one correct answer; (2) Understanding concepts in mathematics is more important than memorizing procedures; (3) It is reasonable to expect people of average mathematical ability to discover some mathematical concepts on their own; and (4) T104 caused preservice teachers to reconsider and, in some cases, change their ideas of how to teach mathematics. Statistically significant improvements were observed for four of five beliefs investigated in a Likert-style beliefs survey. Contains 18 references. (MKR)

THE INFLUENCE OF A PROBLEM-SOLVING APPROACH TO TEACHING MATHEMATICS ON PRESERVICE TEACHERS' MATHEMATICAL BELIEFS

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BACKGROUND

Teachers' Beliefs

Mathematical beliefs do not develop overnight. They develop slowly, over a long period of time involving many mathematical encounters and experiences. Most students' primary source of mathematical experiences is the mathematics classroom. These classroom experiences not only provide the students with the opportunity to learn mathematics, but they also have a strong influence on the students' development of mathematical beliefs; mathematical beliefs that will either help or hinder them as problem solvers (Frank, 1988).

Thompson (1992) and Cobb (1984) have indicated that the fashion in which mathematics is taught conveys a good deal of information about the very nature of mathematics and thereby influences the beliefs students develop about mathematics. Raymond, Santos, and Masingila (1991) state that "teaching actions are directly influenced by teachers' beliefs, and in turn those teacher actions have a tremendous impact on students' belief systems" (p. 2). Hersh (1986) states that "one's conception of what mathematics *is* affects one's conception of how it should be presented" (p. 13).

As evidenced above, what goes on in mathematics classrooms is directly related to the beliefs teachers hold toward mathematics. If healtny mathematical beliefs are to be fostered in students, the teachers' beliefs must also be considered. Many of these beliefs will be the same ones these teachers held as students because beliefs do not change quickly or readily (Owens, 1987; Sowder, 1986). It is interesting to note that Thompson (1984) found that many of the beliefs preservice elementary teachers hold are very similar to severely math-anxious people attending math-anxiety clinics. Ball (1988) and Kogel and Warren (1978) list a number of common teacher beliefs.

It is interesting to note that many of the beliefs teachers have about mathematics are the same beliefs students have or visa versa. Because many teachers "teach as they were taught" (Ball, 1988; Frank, 1990), the beliefs teachers hold are passed on to their students who in turn, when they become teachers, pass them on to their students. Thus the cycle is repeated and the beliefs are instilled in yet another generation of mathematics students. Clearly, if the cycle of unhealthy beliefs about mathematics is going to be broken in this nation, the mathematical beliefs held by preservice and in-service teachers cannot be ignored.

Description of the Class

T104, or *Mathematics for Elementary Teachers Via Problem Solving* as it is officially titled, is an exploratory, student-centered mathematics course that is currently

required of and restricted to preservice elementary and early childhood teachers at Indiana University. This one-semester course is offered by the Department of Mathematics, meets two hours per day, three times per week, and counts as four credits toward graduation. The prerequisite for taking T104 is M118, a course in finite mathematics that is also required of all business majors.

The course uses a problem-solving approach to help preservice teachers gain an understanding of the mathematics they will be required to teach in elementary schools. The areas covered in T104 are numeration, operations, number theory, geometry & measurement, and rational numbers. In T104, understanding involves more than just memorizing mathematical facts and equations and being able to apply them. It includes being able to actually describe, discuss, and apply the underlying concepts in the various areas of mathematics covered.

Since its inception, there have been four main goals for T104.

1. To help students develop adult-level perspectives and insights into the nature of mathematics taught in the elementary school;
2. To improve students' ability to engage in mathematical thinking and reasoning;
3. To increase students' ability to use their mathematical knowledge to solve problems; and
4. To expose students to learning mathematics via problem solving (Lester, 1992, p. 8).

THE STUDY

Overview of the Study

The purpose of this study was to determine the impact T104 actually had on a series of mathematical beliefs held by PSTs. The study also focused on determining if the T104 style of teaching had a stronger influence on challenging the beliefs of students with high or low levels of mathematics achievement. The approach taken to answering the research questions included both quantitative and qualitative methodologies. A Likert-style survey developed to measure the strength of five mathematical beliefs was administered to T104 students from all six sections offered that semester (N=137). The survey was administered in early January and late April. Paired-sample *t*-tests were used to determine if significant changes in any of the beliefs occurred. A short answer survey addressing several beliefs was also given to each student in both January and April. Responses for each student from January and April were compared to look for possible changes in beliefs and trends in changes or lack of change. Additionally, nine PSTs were interviewed in December as they were finishing T104. The interviews focused on identifying changes that might have

occurred in their beliefs about mathematics, and the teaching of mathematics, as well as identifying aspects of T104 that were instrumental in precipitating the changes.

Surveys

The Likert part of the survey contained some questions used by Schoenfeld (1985) and Kloosterman and Stage (1992). Additional questions were written specifically for this survey. The student beliefs that were addressed in the survey are:

a) There is only one way to correctly solve a mathematical problem. (SEVERAL)
b) Step-by-step procedures are needed to solve mathematical problems. (STEP)
c) Mathematics requires mostly memorization. (MEMORY)
d) People of average intelligence are unable to understand mathematics. (UNDERSTAND)
e) If a problem takes more than five to ten minutes it cannot be done.(TIME)

The survey went through several revisions with the final version containing three positively and three negatively worded questions pertaining to each of the beliefs listed above. The survey was administered to PSTs in all six sections of T104. The means, standard deviations, and Cronbach's α obtained from this survey are listed in Table 1 below. The survey was readministered to the same group of students in April.

Table 1
Means, Standard Deviations, and Reliabilities (Cronbach's α) for the Survey (N=137).

Scale	Mean	S.D.	Cronbach's α
SEVERAL	25.37	2.77	.81
STEP	19.23	3.23	.63
MEMORY	21.15	3.80	.74
UNDERSTAND	21.86	3.39	.74
TIME	15.90	4.73	.88

Note. SEVERAL = There is only one way to correctly solve a mathematical problem; STEP = Step-by-step procedures are needed to solve mathematical problems; MEMORY = Mathematics requires mostly memorization; UNDERSTAND = People of average intelligence are unable to understand mathematics; TIME = If a problem takes more than five to ten minutes it cannot be done. Each scale has 6 items and a range of 6 to 30.

Interviews

A total of nine PSTs were interviewed. These were one time interviews that lasted from 45 minutes to an hour each. Each interview was audiotaped and then transcribed. Each interview was semi-structured, consisting of a series of predetermined questions and then also giving the PST an opportunity to make any commment or observations deemed appropriate. The main goal of the interviews was to identify particular facets of T104 that were instrumental in precipitating change in beliefs.

THE RESULTS

Survey Results

Surveys were administered in early January and late April. The data were analyzed to determine if there were significant changes in scores from January to April for each of the five belief scales. The results of the paired sample *t*-tests are found in Table 2.

The *t*-test results in Table 2 indicate positive shifts on each of the belief scales although the shift was not statistically significant in the TIME scale. A mean score of 18

Table 2
Means, Standard Deviations, and Paired Sample t-tests for Changes in Mathematical Beliefs From the Beginning to the End of T104 (N=137).

	Beginning of T104 (January, 1993)		End of T104 (April, 1993)		
Scale	Mean	SD	Mean	SD	*t*
SEVERAL	25.08	2.67	25.66	2.46	2.61*
STEP	19.13	3.27	20.58	3.08	4.81*
MEMORY	21.03	3.75	22.07	3.76	3.67*
UNDER	21.85	3.26	22.81	3.18	3.29*
TIME	15.93	4.61	15.96	4.16	0.13

Note. SEVERAL = There is only one way to correctly solve a mathematical problem; STEP = Step-by-step procedures are needed to solve mathematical problems; MEMORY = Mathematics requires mostly memorization; UNDER -= People of average intelligence are unable to understand mathematics; TIME = If a problem takes more than five to ten minutes it cannot be done.

*p<.01.

for a scale can be interpreted as "undecided" and scores higher than 18 indicate increasingly stronger agreement with a "healthier" position for a particular belief. The highest score possible for any scale is 30 and the lowest is 6.

The students demonstrated a statistically significant increase on the SEVERAL, STEP, MEMORY, and UNDERSTAND scales. These increases suggest positive shifts in the PSTs' mathematical beliefs. There was no significant change in the last scale, TIME. The fact that there was no significant change in the TIME scale suggests that most of the students still held to the notion that no more than five to ten minutes should be spent on any mathematics problem. Further support for this was found in the short answer responses to the question, "If you understand the material, how long should it take to solve a typical homework problem?" The mean time given in January was 9.26 minutes (SD 9.36; N=99). In April the mean time was 11.55 minutes with a standard deviation of 25.19 (N=98). Clearly there was little change in PSTs' thoughts on how long should be spent on solving a mathematics problem.

Interview Results

Careful examination of the interview transcripts resulted in the identification of a number of themes. Some of these themes are directly related to specific questions on the interview and some are the result of additional comments provided by the interviewees. Although there were many more, four key themes are listed below.

- There is more than one way to solve a problem and some problems have more than one correct answer.
- Understanding concepts in mathematics is more important than memorizing procedures.
- It is reasonable to expect people of average mathematical ability to discover some mathematical concepts on their own.
- T104 caused the PSTs to reconsider and, in some cases, change their ideas of how to teach mathematics.

When asked what aspect or aspects of T104 led to either a change in or strengthening of their beliefs, three points were identified. The first was the group work. Working in a group allowed the students to see, firsthand, people taking different approaches to the same problem. A second point that was brought up by two of the students was the exercise in the Geometry/Measurement chapter where the class is asked to develop five different proofs of the Pythagorean Theorem. A third aspect that was mentioned by one person was a required reading, an excerpt from Rebecca Brown Corwin's (1989) article *Multiplication as Original Sin*. She indicated that this article really helped her to see the importance of being open to alternate methods of solution as a teacher.

CONCLUSIONS

Several conclusions seem warranted based on the data presented. The first conclusion is that a problem-solving approach to teaching mathematics to PSTs does have a positive impact, in most cases, on their mathematical beliefs. Statistically significant ($p<.01$) improvements were observed for four of the beliefs when the population was considered as a whole. The population showed a mean change of 0.58 in the scale value in the belief that there is only one way to correctly solve a particular mathematics problem (SEVERAL). A mean increase of 1.46 was observed in the scale value for the belief that step-by-step procedures and algorithms are needed to solve mathematics problems (STEP) and a mean change of 1.04 was obtained for the scale value pertaining to the belief that memorization is the key to success in mathematics (MEMORY). Lastly, an increase of 0.96 was observed in the scale value for the belief that only very intelligent people or geniuses are capable of understanding mathematics (UNDERSTAND). This finding supports Frank's (1990), Buerk's (1985), and Wheatley's (1984) suspicions that a problem-solving, small group approach to teaching mathematics would have a positive impact on students' mathematical beliefs.

A second goal of this study was to determine if the presentation of mathematics from a problem-solving perspective would have a greater impact on high- or low-achieving PSTs. Analysis the data by achievement level indicates that the high-achieving PSTs were more strongly influenced than low-achieving PSTs. Statistically significant changes ($p<.005$) in the mean scale values for SEVERAL, STEP, and MEMORY were observed for PSTs who earned an A in the course. PSTs who earned a B in the course were found to have statistically significant changes ($p<.005$) in the mean scale values for STEP, MEMORY, and UNDERSTAND. No significant changes in the mean scale values were observed for any belief in either the C or D/F achievement level. The data suggest that this problem-solving approach to teaching mathematics has a positive influence on the mathematical beliefs of high-achieving PSTs, but no influence on the beliefs of low achieving PSTs. While the changes did not achieve statistical significance, it is interesting to note that PSTs in the C level had negative changes in the mean scale values for SEVERAL and STEP and the D/F level PSTs had negative changes in the mean scale values for SEVERAL and MEMORY.

The third research question that was addressed in this study focused on identifying those aspects of T104 that were instrumental in challenging the PSTs' beliefs. The PST interviews led to the identification of two particularly important facets of the course. One of these was the group work and the other was the personally experience of regularly working with the problems in T104. The group work allowed the PSTs to see a variety of methods

of solution from others in the group. The group work also allowed the PSTs to occasionally see the limitations of merely having a formula memorized, but not having a real understanding of the underlying concepts.

Regularly working mathematics problems in T104 that conflicted with their beliefs was also important to challenging the PSTs' beliefs. Essentially, the PSTs' regular exposure to mathematics problems that conflicted with their existing beliefs had a significant impact in challenging their beliefs. Personally experiencing situations that conflicted with their established beliefs was an essential part of all of the PSTs' discussions when focusing on identifying facets of the course that were most influential in promoting change. The finding that both personally experiencing mathematical situations that conflict with the students' beliefs and small-group work are important in challenging students' mathematical beliefs further confirms similar observations made by Buerk (1985) and Schram et al. (1988).

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